Updated September 2, 2026

Homework problems for AMAT 540A (Topology I), Fall 2026. Over the course of the semester I'll add problems to this list, with each problem's due date specified. Each problem is worth 2 points.

Solutions will be gradually added (and may be hastily written without proofreading).

Problem 1 (due Weds 9/2): Let $f\colon A \to B$ be a function. Suppose that for all $A_0 \subseteq A$, $f^{-1}(f(A_0)) = A_0$. Prove that f is injective.

Solution: Let $a, a' \in A$ with $f(a) = f(a')$. Set $A_0 = \{a\}$, so $a' \in f^{-1}(f(A_0))$. By hypothesis $a' \in A_0$, so $a = a'$. $\square$

Problem 2 (due Weds 9/2): Let A be a set and let $p_1\colon A \times A \to A$ be the projection function $p_1(a, a') = a$. Let $R \subseteq A \times A$ be a relation such that the image $p_1(R)$ equals all of A. Prove that if R is symmetric and transitive then it is reflexive.

Solution: Suppose R is symmetric and transitive. Let $a \in A$. Since $p_1(R) = A$ there exists $(a_1, a_2) \in R$ with $p(a_1, a_2) = a$. By construction of p this means $a_1 = a$, so a is related to a_2 under R. By symmetry a_2 is related to a, and by transitivity a is related to a, so we conclude R is reflexive. $\square$

Problem 3 (due Weds 9/2): Let $\mathcal{A}$ be a collection of sets and let $U = \bigcup_{A \in \mathcal{A}} A$. Prove that if $\mathcal{P}(U) = \bigcup_{A \in \mathcal{A}} \mathcal{P}(A)$ then $U \in \mathcal{A}$.

Solution: Since $U \in \mathcal{P}(U)$, our hypothesis ensures that $U \in \bigcup_{A \in \mathcal{A}} \mathcal{P}(A)$, i.e., $U \in \mathcal{P}(A)$ for some $A \in \mathcal{A}$. This means $U \subseteq A$, and $A \subseteq U$ by definition, so we conclude $U = A \in \mathcal{A}$ as desired. $\square$

No class Monday 9/7 (Labor Day)

Problem 4 (due Weds 9/9): Prove that $\{U \subseteq \mathbb{N} \mid U = \emptyset \text{ or } |\mathbb{N} \setminus U| < 2026\}$ is not a topology on $\mathbb{N}$.

Problem 5 (due Weds 9/9): Let X be a set and $\sim$ an equivalence relation. Write $[x]$ for the equivalence class of $x \in X$ with respect to $\sim$. Prove that $\mathcal{T} = \{[x] \mid x \in X\}$ is a basis for a topology on X.

Problem 6 (due Weds 9/9): Let $\mathcal{T} = \{U \subseteq \mathbb{R} \mid U = \emptyset \text{ or } \mathbb{R} \setminus U \text{ is countable}\}$. Is $\mathcal{T}$ a topology on $\mathbb{R}$? Prove or disprove.

Problem 7 (due Weds 9/16):

Problem 8 (due Weds 9/16):

Problem 9 (due Weds 9/16):
